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Process and device for determining a transforming element for a given transformation function, method and device for transforming a digital signal from the time domain into the frequency domain and vice versa and computer readable medium

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Cross Reference to Related Applications

This application claims the benefit of priority of U.S. Provisional Application no 60/507,210, filed 29 September 2003, and U.S. Provisional Application no 60/507,440, filed
10   29 September 2003, the contents of each being hereby incorporated by reference in its entirety for all purposes.

Further, the following commonly-owned applications are concurrently-filed herewith, and herein incorporated in its
15   entirety:
        "Method for Performing a Domain Transformation of a Digital Signal from the Time Domain into the Frequency Domain and Vice Versa," Atty. Docket No. P100442, and
        "Method for Transforming a Digital Signal form the Time
20   Domain into the Frequency Domain and Vice Versa," Atty.
        Docket no. P100444.

This invention relates to a process and a device for determining a transforming element for a given transformation
25   function, a method and a device for transforming a digital signal from the time domain into the frequency domain and vice versa and a computer readable medium.

Domain transformations, for example the discrete cosine
30   transform (DCT), are widely used in modern signal processing industry. Recently, a variant of the DCT, called integer DCT, has attracted a lot of research interests because of its important role in lossless coding applications. The term

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"lossless" means that the decoder can generate an exact copy
of the source signal from the encoded bit-stream.

The DCT is a real-valued block transform. Even if the input
5   block consists only of integers, the output block of the DCT
can comprise non-integer components. For convenience, the
input block is referred to as input vector and the output
block as output vector. If a vector comprises only integer
components, it is called an integer vector. In contrast to
10   the DCT, the integer DCT generates an integer output vector
from an integer input vector. For the same integer input
vector, the integer output vector of integer DCT closely
approximates the real output vector of DCT. Thus the integer
DCT keeps all the good properties of the DCT in spectrum
15   analysis.

An important property of the integer DCT is reversibility.
Reversibility means that there exists an integer inverse DCT
(IDCT) so that if the integer DCT generates an output vector
20    $\underline{y}$ from an input vector $\underline{x}$, the integer IDCT can recover the
vector $\underline{x}$ from the vector $\underline{y}$. Sometimes the integer DCT is also
referred to as the forward transform, and the integer IDCT as
the backward or inverse transform.

25   A transform called integer modified discrete cosine transform
(IntMDCT) is recently proposed and used in the ISO/IEC MPEG-4
audio compression. The IntMDCT can be derived from its
prototype - the modified discrete cosine transform (MDCT). In
[1], Malvar gave an efficient realization of MDCT by
30   cascading a bank of Givens rotations with a DCT-IV block. It
is well known that Givens rotation can be factorised into
three lifting steps for mapping integers to integers, see for
example [2].

Therefore, the realization of IntMDCT relies on an efficient
implementation of integer DCT-IV.

5   Integer transforms can be directly converted from their
prototypes by replacing each Givens rotation with three
lifting steps. Because in each lifting step there is one
rounding operation, the total rounding number of an integer
transform is three times the Givens rotation number of the
10   prototype transform. For discrete trigonometric transforms
(for example the Discrete Fourier Transform (DFT) or the
Discrete Cosine Transform (DCT)), the number of Givens
rotations involved is usually at $N\log_2 N$ level, where N is
the size of the blocks, i.e. the amount of data symbols
15   included in each block, the digital signal is divided into.
Accordingly, the total rounding number is also at $N\log_2 N$
level for the family of directly converted integer
transforms. Because of the roundings, an integer transform
only approximates its floating-point prototype. The
20   approximation error increases with the number of rounding
operations.

The invention solves the problem of determining a
transforming element for a given transformation function,
25   which transformation function comprises a transformation
matrix and corresponds to a transformation of a digital
signal from the time domain into the frequency domain or vice
versa, such that the number of roundings comprised by the
transforming element is significantly reduced. The invention
30   further provides a method for transforming a digital signal
from the time domain into the frequency domain or vice versa
according to the determined transforming element.

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The problem is solved by a process and a device for
determining a transforming element for a given transformation
function, a method and a device for transforming a digital
signal from the time domain into the frequency domain or vice

5   versa and a computer readable medium with the features
according to the independent claims.

According to the invention, there is provided a process for
determining a transforming element for a given transformation

10   function, which transformation function comprises a
transformation matrix and corresponds to a transformation of
a digital signal from the time domain into the frequency
domain or vice versa, wherein the transformation matrix is
decomposed into a rotation matrix and an auxiliary matrix

15   which, when multiplied with itself, equals a permutation
matrix multiplied with an integer diagonal matrix; the
rotation matrix and the auxiliary matrix are each decomposed
into a plurality of lifting matrices; and the transforming
element is determined to comprise of a plurality of lifting

20   stages which correspond to the lifting matrices.

Further, according to the invention there is provided a
device adapted to perform the process described above.

25   Further, according to the invention, there is provided a
method for transforming a digital signal from the time domain
into the frequency domain or vice versa using a transforming
element, wherein the transforming element corresponds to a
given transformation function, which transformation function

30   comprises a transformation matrix wherein the transforming
element is determined by a process comprising decomposing the
transformation matrix into a rotation matrix and an auxiliary
matrix which, when multiplied with itself, equals a

permutation matrix multiplied with an integer diagonal
matrix; decomposing the rotation matrix and the auxiliary
matrix each into a plurality of lifting matrices; and
determining the transforming element to comprise of a
5   plurality of lifting stages which correspond to the lifting
matrices; and wherein further each lifting stage comprises
the processing of sub-blocks of the digital signal by an
auxiliary transformation and by a rounding unit.

10   Further, according to the invention there is provided a
device adapted to perform the method described above.

Further, according to the invention, there is provided a
computer readable medium having a program recorded thereon,
15   wherein the program is adapted to make a computer perform a
process for determining a transforming element for a given
transformation function, which transformation function
comprises a transformation matrix and corresponds to a
transformation of a digital signal from the time domain into
20   the frequency domain or vice versa, wherein the
transformation matrix is decomposed into a rotation matrix
and an auxiliary matrix which, when multiplied with itself,
equals a permutation matrix multiplied with an integer
diagonal matrix; the rotation matrix and the auxiliary matrix
25   are each decomposed into a plurality of lifting matrices; and
the transforming element is determined to comprise of a
plurality of lifting stages which correspond to the lifting
matrices.

30   Further, according to the invention, there is provided a
computer readable medium having a program recorded thereon,
wherein the program is adapted to make a computer perform a
method for transforming a digital signal from the time domain

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into the frequency domain or vice versa using a transforming
element, wherein the transforming element corresponds to a
given transformation function, which transformation function
comprises a transformation matrix wherein the transforming

5   element is determined by a process comprising decomposing the
transformation matrix into a rotation matrix and an auxiliary
matrix which, when multiplied with itself, equals a
permutation matrix multiplied with an integer diagonal
matrix; decomposing the rotation matrix and the auxiliary

10   matrix each into a plurality of lifting matrices; determining
the transforming element to comprise of a plurality of
lifting stages which correspond to the lifting matrices; and
wherein further each lifting stage comprises the processing
of sub-blocks of the digital signal by auxiliary

15   transformations and by a rounding unit.

In a preferred embodiment, the invention provides a process
and a method for realizing the integer type-IV DCT
transformation. The method according to the invention

20   requires a significantly reduced number of roundings compared
to prior art methods. As a result, the approximation error is
greatly reduced, in the case of DCT-IV it can be reduced from
the usual $N\log_2 N$ level to be as low as $2.5N$, where N denotes
the block size of the digital signal. The method according to

25   the invention is low in computational complexity and modular
in structure.

The method and the device according to the invention can be
used for any types of digital signals, such as audio, image

30   or video signals. The digital signal, which is a digitised
signal, corresponds to a physical measured signal, may be
generated by scanning at least one characteristic feature of
a corresponding analog signal (for example, the luminance and

chrominance values of a video signal, the amplitude of an analog sound signal, or the analog sensing signal from a sensor). The digital signal comprises a plurality of data symbols. The data symbols of the digital signal are grouped
5   into blocks, with each block having the same predefined number of data symbols based on the sampling rate of the corresponding analog signal.

The method according to the invention can be used for
10 transforming an input digital signal which represents integer values to an output signal which also represents integer values. The transformation method according to the invention is reversible. The output signal may be transformed back to the original input signal by performing the transformation
15 method according to the invention. Such a reversibility property of the transformation according to the method of the invention can be used in lossless coding in which the output signal should be identical to the original input signal.

20 Such an integer transformation of signals according to the invention can be used in many applications and systems such as MPEG audio, image and video compression, JPEG2000 or spectral analyzers (for analyzing Infrared, Ultra-violet or Nuclear Magnetic Radiation signals). It can also be easily
25 implemented in hardware systems such as in a fixed-point Digital Signal Processor (DSP), without having to consider factors such as overflow in the case of a real-value signal transformation.

30 According to the method according to the invention, the digital signal is transformed to the frequency domain by a transforming element, which is according to the process

according to the invention determined for a given
transformation function.

Preferably, the transforming element comprises a plurality of
5   lifting stages.

The transforming element can be illustrated based on the
model of a lifting ladder. The lifting ladder model has two
side pieces, each for receiving one of two groups of data
10   symbols. Two or more cascading lifting stages are provided
between the two side pieces. Each lifting stage receives a
signal at one end (input end), and outputs a signal at the
other end (output end) via a summation unit. A rounding unit
is arranged at the output end. The lifting stages are
15   arranged between the side pieces in an alternating manner,
such that the output (or input) ends of adjacent lifting
stages are connected to the different side pieces.

It should be noted that although the transforming element is
20   described in the form of the lifting ladder model, it is only
to illustrate the transformation paths of the transforming
element. However, the invention shall not be limited to said
ladder model.

25   The number of lifting stages of the transforming element is
defined by the number of lifting matrices which is determined
by the process according to the invention.

Discrete cosine transforms, discrete sine transforms,
30   discrete Fourier transforms or discrete W transforms are
examples of transformation functions that may be used as the
transformation function according to the invention. The
number of lifting stages of the transforming element may be

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different, depending on the result of the process according
to the invention for determining a transforming element for
the respective transformation functions.

5   Illustrative embodiments of the invention are explained below
with reference to the drawings, in which

Figure 1 shows the architecture of an audio encoder according
to an embodiment of the invention.

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Figure 2 shows the architecture of an audio decoder according
to an embodiment of the invention, which corresponds to the
audio coder shown in figure 1.

15   Figure 3 illustrates an embodiment of the process according
to the invention, wherein the transformation function is a
DCT-IV transformation function.

Figure 4 shows a flow chart of an embodiment of the method
20   according to the invention.

Figure 5 illustrates an embodiment of the method according to
the invention using DCT-IV as the transformation function.

25   Figure 6 illustrates the algorithm for the reverse
transformation according to the embodiment of the method
according to the invention illustrated in figure 5.

Figure 7 shows the architecture for an image archiving system
30   according to an embodiment of the invention.

Figure 8 illustrates an embodiment of the method according to
the invention using DWT-IV as the transformation function.

Figure 9 illustrates the algorithm for the reverse
transformation according to the embodiment of the method
according to the invention illustrated in figure 8.

Figure 1 shows the architecture of an audio encoder 100
according to an embodiment of the invention.

The audio encoder 100 comprises a conventional perceptual
base layer coder based on the modified discrete cosine
transform (MDCT) and a lossless enhancement coder based on
the integer modified discrete cosine transform (IntMDCT).

An audio signal 109 which, for instance, is provided by a
microphone 110 and which is digitalized by a analog-to-
digital converter 111 is provided to the audio encoder 100.
The audio signal 109 comprises a plurality of data symbols.
The audio signal 109 is divided into a plurality of blocks,
wherein each block comprises a plurality of data symbols of
the digital signal and each block is transformed by a
modified discrete cosine transform (MDCT) device 101. The
MDCT coefficients provided by the MDCT device 101 are
quantized by a quantizer 103 with the help of a perceptual
model 102. The perceptual model controls the quantizer 103 in
such a way that the audible distortions resulting from the
quantization error are low. The output of the quantizer 103
is encoded by a bitstream encoder 104 which produces the
lossy perceptually coded output bitstream 112. The bitstream
encoder 104 losslessly compresses its input to produce an
output which has a lower average bit-rate than its input by
standard methods such as Huffman-Coding or Run-Length-Coding.
The input audio signal 109 is also fed into an IntMDCT device
105 which produces IntMDCT coefficients. The quantized MDCT

coefficients, which are the output of the quantizer 103, are
used to predict the IntMDCT coefficients. The quantized MDCT
coefficients are fed into an inverse quantizer 106 and the
output of the inverse quantizer 106 is fed into a rounding

5   unit 107 by which they are rounded to integer values and the
residual IntMDCT coefficients, which are the difference
between the output of the rounding unit 107 and the IntMDCT
coefficients, are entropy coded by an entropy coder 108,
which, analogous to the bitstream encoder 104, losslessly

10   reduces the average bit-rate of its input and produces a
lossless enhancement bitstream 113, which, together with the
perceptually coded bitstream 112, carries the necessary
information to reconstruct the input audio signal 109
exactly.

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Figure 2 shows the architecture of an audio decoder 200
comprising an embodiment of the invention, which corresponds
to the audio coder 100 shown in figure 1.

20   The perceptually coded bitstream 207 is decoded by a
bitstream decoder 201, which performs the inverse operations
to the operations of the bitstream encoder 104 of figure 1
and is fed to an inverse quantizer 202. To its output, the
inverse MDCT is applied by an inverse MDCT device 203. Thus,

25   the reconstructed perceptually coded audio signal 209 is
obtained. The lossless enhancement bitstream 208 is decoded
by an entropy decoder 204, which performs the inverse
operations to the operations of the entropy encoder 108 of
figure 1 and which produces the corresponding residual

30   IntMDCT coefficients. The output of the inverse quantizer 202
is rounded by a rounding device 205 and is added to the
residual IntMDCT coefficients, thus producing the IntMDCT
coefficients. Finally, the inverse IntMDCT is applied to the

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IntMDCT coefficients by an inverse IntMDCT device 206 to
produce the reconstructed losslessly coded audio signal 210.

As mentioned above, it is shown in [2] that the core of

5   IntMDCT, which plays an important role in lossless audio
coding and which is used in the embodiment of the invention
illustrated in figures 1 and 2, is an integer DCT-IV.

Figure 3 illustrates a flowchart of an embodiment of the

10   process according to the invention, wherein the
transformation function is a DCT-IV transformation function.

In the following, an embodiment of the process according to
the invention for determining a transforming element for a

15   DCT-IV transformation function is explained. The determined
transforming element is used in the encoder shown in figure 1
to implement IntMDCT and the according inverse transforming
element is used in the decoder shown in figure 2 to implement
inverse IntMDCT. For a description how to implement IntMDCT

20   and inverse IntMDCT with DCT-IV, see [2].

The DCT-IV transformation function with a N-point real input
sequence $x(n)$ is defined as follows (see [2]):

25   $$y(m) = \sqrt{\frac{2}{N}} \sum_{n=0}^{N-1} x(n) \cos\left(\frac{(m+1/2)(n+1/2)\pi}{N}\right) \quad m,n = 0,1,\cdots,N-1 \qquad (1)$$

Let $\underline{C_N^{IV}}$ be the transformation matrix of DCT-IV, that is

$$\underline{C_N^{IV}} = \sqrt{\frac{2}{N}} \left[\cos\left(\frac{(m+1/2)(n+1/2)\pi}{N}\right)\right]_{m,n = 0,1,\cdots,N-1} \qquad (2)$$

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According to the embodiment of process according to the invention, the transformation matrix C_N^{IV} is decomposed into a rotation matrix and a matrix which, when multiplied with itself, equals a permutation matrix multiplied with an integer diagonal matrix.

For clarity, N is assumed to be even.

In step 300, the process is started.

In step 301 the terms of the transformation function for even indices are separated from the terms with odd indices:

$$y(m) = \sqrt{\frac{2}{N}} \sum_{n=0}^{N-1} x(n) \cos\left((m+\frac{1}{2})(n+\frac{1}{2})\frac{\pi}{N}\right)$$

$$= \sqrt{\frac{2}{N}} \sum_{n\,even} x(n) \cos\left((m+\frac{1}{2})(n+\frac{1}{2})\frac{\pi}{N}\right) + \sqrt{\frac{2}{N}} \sum_{n\,odd} x(n) \cos\left((m+\frac{1}{2})(n+\frac{1}{2})\frac{\pi}{N}\right)$$

$$= \sqrt{\frac{2}{N}} \sum_{n=0}^{\frac{N}{2}-1} x(2n) \cos\left((m+\frac{1}{2})(2n+\frac{1}{2})\frac{\pi}{N}\right) + \sqrt{\frac{2}{N}} \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) \cos\left((m+\frac{1}{2})(2n+1+\frac{1}{2})\frac{\pi}{N}\right)$$

$$= \sqrt{\frac{2}{N}} \sum_{n=0}^{\frac{N}{2}-1} x_1(n) \cos\left((m+\frac{1}{2})(n+\frac{1}{4})\frac{\pi}{\left(\frac{N}{2}\right)}\right) + \sqrt{\frac{2}{N}} \sum_{n=0}^{\frac{N}{2}-1} x_2(n) \cos\left((m+\frac{1}{2})(n+\frac{3}{4})\frac{\pi}{\left(\frac{N}{2}\right)}\right) ;$$

thus,

$$y(m) = \sqrt{\frac{2}{N}} \sum_{n=0}^{\frac{N}{2}-1} x_1(n) \cos\left((m+\frac{1}{2})(n+\frac{1}{2})\frac{\pi}{\left(\frac{N}{2}\right)} - \frac{1}{4}(m+\frac{1}{2})\frac{\pi}{\left(\frac{N}{2}\right)}\right)$$

$$+ \sqrt{\frac{2}{N}} \sum_{n=0}^{\frac{N}{2}-1} x_2(n) \cos\left((m+\frac{1}{2})(n+\frac{1}{2})\frac{\pi}{\left(\frac{N}{2}\right)} + \frac{1}{4}(m+\frac{1}{2})\frac{\pi}{\left(\frac{N}{2}\right)}\right) \qquad (3)$$

where $\underline{x_1}$ with the components $x_1(n) = x(2n)$ $,n = 0,...,\dfrac{N}{2}-1$ is a

vector comprising all $x(n)$ with even indices and $\underline{x_2}$ with the

components $x_2(n) = x(2n+1)$ $,n = 0,...,\dfrac{N}{2}-1$ is a vector comprising

5   all $x(n)$ with odd indices.

The following two abbreviations are used:

$$\alpha_{m,n} = (m+\frac{1}{2})(n+\frac{1}{2})\frac{\pi}{\left(\dfrac{N}{2}\right)} \quad ,m,n = 0,...,N-1 \tag{4}$$

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$$\beta_m = \frac{1}{4}(m+\frac{1}{2})\frac{\pi}{\left(\dfrac{N}{2}\right)} = (m+\frac{1}{2})\frac{\pi}{2N} \quad ,m = 0,...,N-1 \tag{5}$$

With equations (4) and (5), equation (3) can be written as

15   $$y(m) = \sqrt{\frac{2}{N}}\sum_{n=0}^{\frac{N}{2}-1} x_1(n)\cos(\alpha_{m,n} - \beta_m) + \sqrt{\frac{2}{N}}\sum_{n=0}^{\frac{N}{2}-1} x_2(n)\cos(\alpha_{m,n} + \beta_m) \tag{6}$$

In step 302, the following two addition theorems for the
cosine are applied:

20   $$\cos(\alpha_{m,n} - \beta_m) = \cos\alpha_{m,n} \cdot \cos\beta_m + \sin\alpha_{m,n} \cdot \sin\beta_m \tag{7}$$

$$\cos(\alpha_{m,n} + \beta_m) = \cos\alpha_{m,n} \cdot \cos\beta_{m,n} - \sin\alpha_{m,n} \cdot \sin\beta_m \tag{8}$$

With (7) and (8), equation (6) can be written as

$$y(m) = \sqrt{\frac{2}{N}}\left\{\sum_{n=0}^{\frac{N}{2}-1}x_1(n)\cos\alpha_{m,n}\cos\beta_m + \sum_{n=0}^{\frac{N}{2}-1}x_1(n)\sin\alpha_{m,n}\sin\beta_m + \sum_{n=0}^{\frac{N}{2}-1}x_2(n)\cos\alpha_{m,n}\cos\beta_m - \sum_{n=0}^{\frac{N}{2}-1}x_2(n)\sin\alpha_{m,n}\sin\beta_m\right\}$$

$$= \sqrt{\frac{2}{N}}\left\{\sum_{n=0}^{\frac{N}{2}-1}[x_1(n)\cos\beta_m + x_2(n)\cos\beta_m]\cos\alpha_{m,n} + \sum_{n=0}^{\frac{N}{2}-1}[x_1(n)\sin\beta_m - x_2(n)\sin\beta_m]\sin\alpha_{m,n}\right\}$$

$$= \sqrt{\frac{2}{\left(\frac{N}{2}\right)}}\left\{\sum_{n=0}^{\frac{N}{2}-1}\frac{\cos\beta_m}{\sqrt{2}}[x_1(n) + x_2(n)]\cos\alpha_{m,n} + \sum_{n=0}^{\frac{N}{2}-1}\frac{\sin\beta_m}{\sqrt{2}}[x_1(n) - x_2(n)]\sin\alpha_{m,n}\right\} \qquad (9)$$

for abbreviation, the two N/2-dimensional vectors $\underline{x_1}'$ and $\underline{x_2}'$ are defined to comprise the following components:

$$x_1'(n) = \frac{1}{\sqrt{2}}[x_1(n) + x_2(n)] \quad ,n = 0,\ldots,\frac{N}{2}-1 \qquad (10)$$

$$x_2'(n) = \frac{1}{\sqrt{2}}[x_1(n) - x_2(n)] \quad ,n = 0,\ldots,\frac{N}{2}-1 \qquad (11)$$

With (10) and (11), equation (9) simplifies to

$$y(m) = \sqrt{\frac{2}{\left(\frac{N}{2}\right)}}\left\{\sum_{n=0}^{\frac{N}{2}-1}\cos\beta_m x_1'(n)\cos\alpha_{m,n} + \sum_{n=0}^{\frac{N}{2}-1}\sin\beta_m x_2'(n)\sin\alpha_{m,n}\right\} \qquad (12)$$

In step 303, the vector

$$\underline{y} = \begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(N) \end{bmatrix}$$

is divided into the two parts $\underline{y_0}$ and $\underline{y_1}$ wherein

$$
\underline{y}_0 = \begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(\frac{N}{2}-1) \end{bmatrix} \tag{13}
$$

$$
\underline{y}_1 = \begin{bmatrix} y(N-1) \\ y(N-2) \\ \vdots \\ y(\frac{N}{2}) \end{bmatrix} \tag{14}
$$

The vector $\underline{y}_1$ comprises the components of $\underline{y}$ corresponding to the indices from $\dfrac{N}{2}$ to $N-1$ in reverse order.

The components $y_1(m)$ of the vector $\underline{y}_1$ fulfil the following equation:

$$
\begin{aligned}
y_1(m) &= y(N-1-m) \\
&= \sqrt{\frac{2}{\left(\frac{N}{2}\right)}} \left\{ \sum_{n=0}^{\frac{N}{2}-1} \cos \beta_{N-1-m} x_1{}'(n) \cos \alpha_{N-1-m,n} + \sum_{n=0}^{\frac{N}{2}-1} \sin \beta_{N-1-m,m} x_2{}'(n) \sin \alpha_{N-1-m,n} \right\}
\end{aligned} \tag{15}
$$

for $m = 0,\ldots,\dfrac{N}{2}-1$.

Note that

$$
y(m+\frac{N}{2}) = \sqrt{\frac{2}{\left(\frac{N}{2}\right)}} \left\{ \sum_{n=0}^{\frac{N}{2}-1} \cos \beta_{m+\frac{N}{2}} x_1{}'(n) \cos \alpha_{m+\frac{N}{2},n} + \sum_{n=0}^{\frac{N}{2}-1} \sin \beta_{m+\frac{N}{2}} x_2{}'(n) \sin \alpha_{m+\frac{N}{2},n} \right\} \tag{16}
$$

for $m = 0,\ldots,\dfrac{N}{2}-1$.

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In step 304, $\underline{y}_0$ and $\underline{y}_1$ are each expressed by a DCT-IV matrix $C_{\frac{N}{2}}^{IV}$ and a DST-IV matrix $S_{\frac{N}{2}}^{IV}$, each of dimension $\frac{N}{2}$.

5   This is achieved in the following way:

For $\beta_m = (m + \frac{1}{2})\frac{\pi}{2N}$ the following equations hold:

$$\beta_{m+\frac{N}{2}} = (m + \frac{1}{2} + \frac{N}{2})\frac{\pi}{2N} = (m + \frac{1}{2})\frac{\pi}{2N} + \frac{N}{2} \cdot \frac{\pi}{2N} = \beta_m + \frac{\pi}{4} \qquad (17)$$

10   $$\beta_{N-1-m} = (N - 1 - m + \frac{1}{2})\frac{\pi}{2N} = N \cdot \frac{\pi}{2N} - (m + \frac{1}{2}) \cdot \frac{\pi}{2N} = \frac{\pi}{2} - \beta_m \qquad (18)$$

Further,

$$\cos\beta_{N-1-m} = \cos(\frac{\pi}{2} - \beta_m) = \sin\beta_m \qquad (19)$$

15   $$\sin\beta_{N-1-m} = \sin(\frac{\pi}{2} - \beta_m) = \cos\beta_m \qquad (20)$$

For $\alpha_{m,n} = (m + \frac{1}{2})(n + \frac{1}{2})\dfrac{\pi}{\left(\dfrac{N}{2}\right)}$ the following equation holds:

20   $$\alpha_{N-1-m,n} = (N - 1 - m + \frac{1}{2})(n + \frac{1}{2})\frac{\pi}{\left(\frac{N}{2}\right)} = (N - m - \frac{1}{2})(n + \frac{1}{2})\frac{\pi}{\left(\frac{N}{2}\right)}$$

$$= N(n + \frac{1}{2})\frac{\pi}{\left(\frac{N}{2}\right)} - (m + \frac{1}{2})(n + \frac{1}{2})\frac{\pi}{\left(\frac{N}{2}\right)} = 2\pi(n + \frac{1}{2}) - \alpha_{m,n} = 2\pi n + \pi - \alpha_{m,n}$$

$$(21)$$

Further:

$$\cos\alpha_{N-1-m,n} = \cos(2\pi\, n + \pi - \alpha_{m,n}) = \cos(\pi - \alpha_{m,n}) = -\cos\alpha_{m,n} \qquad (22)$$

$$\sin\alpha_{N-1-m,n} = \sin(2\pi\, n + \pi - \alpha_{m,n}) = \sin(\pi - \alpha_{m,n}) = \sin\alpha_{m,n} \qquad (23)$$

5

With the equations (19), (20), (22) and (23) equation (15) can be rewritten:

$$y(N-1-m) = \sqrt{\frac{2}{\left(\frac{N}{2}\right)}}\left\{\sum_{n}^{\frac{N}{2}-1}\cos\beta_{N-1-m}x_1{'}(n)\cos\alpha_{N-1-m,n} + \sum_{n}^{\frac{N}{2}-1}\sin\beta_{N-1-m}x_2{'}(n)\sin\alpha_{N-1-m,n}\right\}$$

10
$$= \sqrt{\frac{2}{\left(\frac{N}{2}\right)}}\left\{\sum_{n}^{\frac{N}{2}-1}\sin\beta_m x_1{'}(n)(-\cos\alpha_{m,n}) + \sum_{n}^{\frac{N}{2}-1}\cos\beta_m x_2{'}(n)\sin\alpha_{m,n}\right\}$$

$$= -\sqrt{\frac{2}{\left(\frac{N}{2}\right)}}\sum_{n}^{\frac{N}{2}-1}\sin\beta_m x_1{'}(n)\cos\alpha_{m,n} + -\sqrt{\frac{2}{\left(\frac{N}{2}\right)}}\sum_{n}^{\frac{N}{2}-1}\cos\beta_m x_2{'}(n)\sin\alpha_{m,n}$$

$$= -\sin\beta_m\left[\sqrt{\frac{2}{\left(\frac{N}{2}\right)}}\sum_{n}^{\frac{N}{2}-1}x_1{'}(n)\cos\alpha_{m,n}\right] + \cos\beta_m\left[\sqrt{\frac{2}{\left(\frac{N}{2}\right)}}\sum_{n}^{\frac{N}{2}-1}x_2{'}(n)\sin\alpha_{m,n}\right] \qquad (24).$$

for $m = 0,...,\dfrac{N}{2}-1$.

15

Since further equation (12) yields

$$y(m) = \sqrt{\frac{2}{\left(\frac{N}{2}\right)}}\left\{\sum_{n}^{\frac{N}{2}-1}\cos\beta_m x_1{'}(n)\cos\alpha_{m,n} + \sum_{n}^{\frac{N}{2}-1}\sin\beta_m x_2{'}(n)\sin\alpha_{m,n}\right\}$$

19

$$= \cos \beta_m \left[\sqrt{\frac{2}{\left(\frac{N}{2}\right)}} \sum_{n}^{\frac{N}{2}-1} x_1'(n) \cos \alpha_{m,n} \right] + \sin \beta_m \left[\sqrt{\frac{2}{\left(\frac{N}{2}\right)}} \sum_{n}^{\frac{N}{2}-1} x_2'(n) \sin \alpha_{m,n} \right] \qquad (25)$$

the expressions for $\underline{y_0}$ and $\underline{y_1}$ can be formed from (24) and (25):

$$\underline{y_0} = diag[\cos \beta_m] \cdot C_{\frac{N}{2}}^{IV} \cdot \underline{x_1}' + diag[\sin \beta_m] \cdot S_{\frac{N}{2}}^{IV} \cdot \underline{x_2}', \qquad m = 0,...,\frac{N}{2} - 1 \qquad (26)$$

$$\underline{y_1} = diag[-\sin \beta_m] \cdot C_{\frac{N}{2}}^{IV} \cdot \underline{x_1}' + diag[\cos \beta_m] \cdot S_{\frac{N}{2}}^{IV} \cdot \underline{x_2}', \qquad m = 0,...,\frac{N}{2} - 1 \qquad (27)$$

where $diag[a_m]$ denotes a N/2-by-N/2 diagonal matrix with a_m in the mth row, $C_{\frac{N}{2}}^{IV}$ is the transformation matrix of the DCT-IV transformation and $S_{\frac{N}{2}}^{IV}$ is the transformation matrix of the discrete sine transformation of type four (DST-IV).

The two equations (26) and (27) can be expressed as a single one in the following way:

$$\begin{bmatrix} y_0(0) \\ \vdots \\ y_0(\frac{N}{2}-1) \\ y_1(0) \\ \vdots \\ y_1(\frac{N}{2}-1) \end{bmatrix} = \begin{bmatrix} \cos \beta_1 & & & \sin \beta_1 & & \\ & \ddots & & & \ddots & \\ & & \cos \beta_m & & & \sin \beta_m \\ -\sin \beta_1 & & & \cos \beta_1 & & \\ & \ddots & & & \ddots & \\ & & -\sin \beta_m & & & \cos \beta_m \end{bmatrix} \begin{bmatrix} C_{\frac{N}{2}}^{IV} & \\ & S_{\frac{N}{2}}^{IV} \end{bmatrix} \begin{bmatrix} x_1'(0) \\ \vdots \\ x_1'(\frac{N}{2}-1) \\ x_2'(0) \\ \vdots \\ x_2'(\frac{N}{2}-1) \end{bmatrix}$$

$$(28)$$

In the following, the $\frac{N}{2} \times \frac{N}{2}$ - matrices

$$I_{\frac{N}{2}} = \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \end{bmatrix} \tag{29}$$

and

$$J_{\frac{N}{2}} = \begin{bmatrix} & & & & 1 \\ & & & 1 & \\ & & 1 & & \\ & \cdot^{\cdot^{\cdot}} & & & \\ 1 & & & & \end{bmatrix} \tag{30}$$

are used.

One sees that

$$\begin{bmatrix} y(0) \\ y(2) \\ \vdots \\ y(N-1) \end{bmatrix} = \begin{bmatrix} I_{\frac{N}{2}} & \\ & J_{\frac{N}{2}} \end{bmatrix} \begin{bmatrix} y_0(0) \\ \vdots \\ y_0(\frac{N}{2}-1) \\ y_1(0) \\ \vdots \\ y_1(\frac{N}{2}-1) \end{bmatrix} \tag{31}$$

which is abbreviated by

$$\underline{y} = \begin{bmatrix} I_{\frac{N}{2}} & \\ & J_{\frac{N}{2}} \end{bmatrix} \begin{bmatrix} \underline{y_0} \\ \underline{y_1} \end{bmatrix} \tag{32}$$

One further sees that with the $N \times N$-matrix $\underline{R}_{pr}$ defined as

$$\underline{R}_{pr} = \frac{1}{\sqrt{2}} \begin{bmatrix} I_{N/2} & I_{N/2} \\ I_{N/2} & -I_{N/2} \end{bmatrix} \tag{33}$$

the equation

5

$$\begin{bmatrix} x_1'(0) \\ \vdots \\ x_1'(\frac{N}{2}-1) \\ x_2'(0) \\ \vdots \\ x_2'(\frac{N}{2}-1) \end{bmatrix} = \underline{R}_{pr} \begin{bmatrix} x_1(0) \\ \vdots \\ x_1(\frac{N}{2}-1) \\ x_2(0) \\ \vdots \\ x_2(\frac{N}{2}-1) \end{bmatrix} \tag{34}$$

holds.

10   Equation (34) is abbreviated by

$$\begin{bmatrix} \underline{x_1'} \\ \underline{x_2'} \end{bmatrix} = \underline{R}_{pr} \begin{bmatrix} \underline{x_1} \\ \underline{x_2} \end{bmatrix} \tag{35}$$

Further, let $\underline{P}_{eo}$ be an even-odd matrix, i.e., a permutation
15   matrix reordering the components of the vector

$$\underline{x} = \begin{bmatrix} x(0) \\ \vdots \\ x(N-1) \end{bmatrix}$$

by separating the components which correspond to even indices
20   from the components which correspond to odd indices, i.e.,
such that

$$\begin{bmatrix} x_1(0) \\ \vdots \\ x_1(\frac{N}{2}-1) \\ x_2(0) \\ \vdots \\ x_2(\frac{N}{2}-1) \end{bmatrix} = \underline{P_{eo}} \begin{bmatrix} x(0) \\ \vdots \\ x(N-1) \end{bmatrix} \tag{36}$$

or shortly

$$5 \qquad \begin{bmatrix} \underline{x_1} \\ \underline{x_2} \end{bmatrix} = \underline{P_{eo}}\,\underline{x} \tag{37}$$

holds.

By combining the equation (28) with the equations (31), (34)
10   and (36) one gets:

$$\begin{bmatrix} y(0) \\ y(2) \\ \vdots \\ y(N-1) \end{bmatrix} = \begin{bmatrix} \underline{I_{\frac{N}{2}}} & \\ & \underline{J_{\frac{N}{2}}} \end{bmatrix} \begin{bmatrix} \cos\beta_1 & & & \sin\beta_1 & & \\ & \ddots & & & \ddots & \\ & & \cos\beta_m & & & \sin\beta_m \\ -\sin\beta_1 & & & \cos\beta_1 & & \\ & \ddots & & & \ddots & \\ & & -.\sin\beta_m & & & \cos\beta_m \end{bmatrix} \begin{bmatrix} \underline{C_{\frac{N}{2}}^{IV}} & \\ & \underline{S_{\frac{N}{2}}^{IV}} \end{bmatrix} \underline{R_{pr}}\,\underline{P_{eo}} \begin{bmatrix} x(0) \\ \vdots \\ x(N-1) \end{bmatrix}$$

$$\tag{38}$$

15

with the above abbreviations and the abbreviations

$$\underline{C} = \begin{bmatrix} \cos\beta_1 & & \\ & \ddots & \\ & & \cos\beta_m \end{bmatrix} \tag{39}$$

20   and

$$\underline{S} = \begin{bmatrix} \sin \beta_1 & & \\ & \ddots & \\ & & \sin \beta_m \end{bmatrix} \tag{40}$$

equation (38) can be shortly written as

$$\underline{y} = \begin{bmatrix} I_{\frac{N}{2}} & \\ & J_{\frac{N}{2}} \end{bmatrix} \begin{bmatrix} \underline{C} & \underline{S} \\ -\underline{S} & \underline{C} \end{bmatrix} \begin{bmatrix} C_{\frac{N}{2}}^{IV} & \\ & S_{\frac{N}{2}}^{IV} \end{bmatrix} R_{pr} P_{eo} \underline{x} \tag{41}$$

In step 305 $S_{\frac{N}{2}}^{IV}$ is expressed by $C_{\frac{N}{2}}^{IV}$. This is done with the

equation:

$$S_{\frac{N}{2}}^{IV} = J_{\frac{N}{2}} \cdot C_{\frac{N}{2}}^{IV} \cdot D_{\frac{N}{2}} \tag{42}$$

wherein $\underline{D_{N/2}}$ is a diagonal matrix of dimension *N/2* given by

$$D_{\frac{N}{2}} = \begin{bmatrix} 1 & & & & \\ & -1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & -1 \end{bmatrix} \tag{43}$$

With equation (42) equation (41) can be written as

$$\underline{y} = \begin{bmatrix} I_{\frac{N}{2}} & \\ & J_{\frac{N}{2}} \end{bmatrix} \begin{bmatrix} \underline{C} & \underline{S} \\ -\underline{S} & \underline{C} \end{bmatrix} \begin{bmatrix} I_{\frac{N}{2}} & \\ & J_{\frac{N}{2}} \end{bmatrix} \begin{bmatrix} C_{\frac{N}{2}}^{IV} & \\ & C_{\frac{N}{2}}^{IV} D_{\frac{N}{2}} \end{bmatrix} R_{pr} P_{eo} \underline{x} \tag{44}$$

24

In step 306 the $N \times N$ rotation matrix $\underline{R}_{po}$ is calculated,

which comprises the first three matrices of equation (44):

$$\underline{R}_{po} = \begin{bmatrix} I_{\frac{N}{2}} & \\ & J_{\frac{N}{2}} \end{bmatrix} \begin{bmatrix} \underline{C} & \underline{S} \\ -\underline{S} & \underline{C} \end{bmatrix} \begin{bmatrix} I_{\frac{N}{2}} & \\ & J_{\frac{N}{2}} \end{bmatrix} \qquad (45)$$

5

Multiplication of the three matrices in equation (45) yields

$$\underline{R}_{po} = \begin{bmatrix} \cos\frac{\pi}{4N} & & & & & & & \sin\frac{\pi}{4N} \\ & \cos\frac{3\pi}{4N} & & & & & \sin\frac{3\pi}{4N} & \\ & & \ddots & & & \cdot^{\cdot^{\cdot}} & & \\ & & & \cos\frac{(N-1)\pi}{4N} & \sin\frac{(N-1)\pi}{4N} & & & \\ & & & -\sin\frac{(N-1)\pi}{4N} & \cos\frac{(N-1)\pi}{4N} & & & \\ & & \cdot^{\cdot^{\cdot}} & & & \ddots & & \\ & -\sin\frac{3\pi}{4N} & & & & & \cos\frac{3\pi}{4N} & \\ -\sin\frac{\pi}{4N} & & & & & & & \cos\frac{\pi}{4N} \end{bmatrix} \qquad (46)$$

10   In step 307, the auxiliary matrix, which, when multiplied
with itself, equals a permutation matrix multiplied with an
integer diagonal matrix, is calculated.

The auxiliary matrix comprises the fourth and fifth matrices

15   of equation (44):

$$\underline{T} = \begin{bmatrix} C_{N/2}^{IV} & \\ & \underline{C}_{N/2}^{IV}\underline{D}_{N/2} \end{bmatrix} \underline{R}_{pr} = \frac{1}{\sqrt{2}} \begin{bmatrix} C_{N/2}^{IV} & C_{N/2}^{IV} \\ \underline{C}_{N/2}^{IV}\underline{D}_{N/2} & -\underline{C}_{N/2}^{IV}\underline{D}_{N/2} \end{bmatrix} \qquad (47)$$

Note that

20

$$
\underline{T} \cdot \underline{T} = \begin{bmatrix} 1 & & & & 0 & & & \\ & 0 & & & & 1 & & \\ & & \ddots & & & & \ddots & \\ & & & 1 & & & & 0 \\ & & & 0 & & & & 1 \\ 0 & & & & 1 & & & \\ & -1 & & & & 0 & & \\ & & \ddots & & & & \ddots & \\ & & & 0 & & & & 1 \\ & & & -1 & & & & 0 \end{bmatrix} = \begin{bmatrix} I_{\frac{N}{2}} & \\ & D_{\frac{N}{2}} \end{bmatrix} \begin{bmatrix} 1 & & & & 0 & & 1 & \\ & 0 & & & & 1 & & \\ & & \ddots & & & & & \ddots \\ & & & 1 & & & & 0 \\ & & & 0 & & & & 1 \\ 0 & & & & 1 & & 0 & \\ & 1 & & & & 0 & & \\ & & \ddots & & & & & \ddots \\ & & & 0 & & & & 1 \\ & & & 1 & & & & 0 \end{bmatrix}
$$

(48)

I.e., $\underline{T}$ multiplied with itself equals a permutation matrix
multiplied with an integer diagonal matrix.

With equations (46) and (47), equation (44) simplifies to

$$\underline{C_N^{IV}} = \underline{R_{po}} \, \underline{T} \, \underline{P_{eo}} \tag{49}$$

Thus, the transformation matrix $\underline{C_N^{IV}}$ is decomposed into a
rotation matrix $\underline{R_{po}}$, into an auxiliary matrix $\underline{T}$ that, when
multiplied with itself, equals a permutation matrix
multiplied with an integer diagonal matrix and into an even-
odd matrix $\underline{P_{eo}}$.

In step 308, $\underline{R_{po}}$ and $\underline{T}$ are each factorised into a product of
lifting matrices. $\underline{T}$ is factorised in the following way:

$$\underline{T} = \underline{T_1 T_2 T_3} = \begin{bmatrix} I_{N/2} & \\ K_1 & I_{N/2} \end{bmatrix} \cdot \begin{bmatrix} -D_{N/2} & K_2 \\ & I_{N/2} \end{bmatrix} \cdot \begin{bmatrix} I_{N/2} & \\ K_3 & I_{N/2} \end{bmatrix} \tag{50}$$

where

$$\underline{K_1} = -\left(\underline{C^{IV}_{N/2}}\underline{D_{N/2}} + \sqrt{2}\underline{I_{N/2}}\right)\underline{C^{IV}_{N/2}} \tag{51}$$

$$\underline{K_2} = \frac{\underline{C^{IV}_{N/2}}}{\sqrt{2}} \tag{52}$$

$$\underline{K_3} = \sqrt{2}\underline{C^{IV}_{N/2}}\underline{D_{N/2}} + \underline{I_{N/2}} \tag{53}$$

5   And $\underline{R_{po}}$ is factorised according to the following equation:

$$\underline{R_{po}} = \underline{R_1}\underline{R_2}\underline{R_3} = \begin{bmatrix} \underline{I_{N/2}} & \\ \underline{H_1} & \underline{I_{N/2}} \end{bmatrix} \cdot \begin{bmatrix} \underline{I_{N/2}} & \underline{H_2} \\ & \underline{I_{N/2}} \end{bmatrix} \cdot \begin{bmatrix} \underline{I_{N/2}} & \\ \underline{H_3} & \underline{I_{N/2}} \end{bmatrix} \tag{54}$$

where

10

$$\underline{H_1} = \underline{H_3} = \begin{bmatrix} & & & -\tan\frac{(N-1)\pi}{8N} \\ & & \ddots & \\ & -\tan\frac{3\pi}{8N} & & \\ -\tan\frac{\pi}{8N} & & & \end{bmatrix} \tag{55}$$

and

15   $$\underline{H_2} = \begin{bmatrix} & & & \sin\frac{\pi}{4N} \\ & & \sin\frac{3\pi}{4N} & \\ & \ddots & & \\ \sin\frac{(N-1)\pi}{4N} & & & \end{bmatrix} \tag{56}$$

Thus, Equation (49) can be re-written as

$$\underline{C^{IV}_N} = \underline{R_1}\underline{R_2}\underline{R_3}\underline{T_1}\underline{T_2}\underline{T_3}\underline{P_{eo}} \tag{57}$$

20

In step 309, the lifting matrices are aggregated as far as possible. Matrix $\underline{S}$ is defined as the product of $\underline{R_3}$ and $\underline{T_1}$, that is

5   $$\underline{S} = \underline{R_3}\underline{T_1} = \begin{bmatrix} I_{N/2} & \\ \underline{H_3} + \underline{K_1} & I_{N/2} \end{bmatrix} \qquad (58)$$

This aggregation is possible since $\underline{S}$ is also a lifting matrix.

10   From (57) and (58), the final factorisation formula for DCT-IV matrix is obtained as

$$\underline{C_N^{IV}} = \underline{R_1}\underline{R_2}\underline{S}\ \underline{T_2}\underline{T_3}\underline{P_{eo}} \qquad (59)$$

15   Equation (59) indicates that the transforming element for the integer DCT-IV transformation according to the invention comprises five lifting stages.

Since the final factorisation formula is determined, the
20   process is stopped in step 310.

Figure 4 shows a flow chart 400 of an embodiment of the method according to the invention using five lifting stages, a first lifting stage 401, a second lifting stage 402, a
25   third lifting stage 403, a fourth lifting stage 404 and a fifth lifting stage 405. This method is preferably used in the IntMDCT device 105 of figure 1 and the inverse IntMDCT device 206 of figure 2 to implement IntMDCT and inverse IntMDCT, respectively. In Fig. 4, $\underline{x_1}$ and $\underline{x_2}$ are first and
30   second blocks of the digital signal, respectively. $\underline{z_1}$, $\underline{z_2}$, $\underline{z_3}$ are intermediate signals, and $\underline{y_1}$ and $\underline{y_2}$ are output signals

corresponding to the first and second block of the digital
signal, respectively.

Figure 5 shows a flow chart of an embodiment of the method
according to the invention, wherein the transformation
function is a DCT-IV transformation function. The
transforming element used in this embodiment corresponds to
the equation (59), i.e., is the one which is determined by
the embodiment of the process illustrated in figure 3.

The transforming element comprises five lifting stages which
correspond to the five lifting matrices of equation (59).

Further, the transforming element comprises a data shuffling
stage corresponding to the permutation matrix $\underline{P}_{eo}$.

In figure 5, the input of the first lifting stage are the two
blocks of the digital signal $\underline{x}_1$ and $\underline{x}_2$, $\underline{z}_1$, $\underline{z}_2$ and $\underline{z}_3$ are
intermediate signals and $\underline{y}_1$ and $\underline{y}_2$ are output signals
corresponding to the first and second block of the digital
signal, respectively.

The input of the transforming element $\underline{x}$ and the two input
blocks of the first lifting stage of the transforming element
$\underline{x}_1$ and $\underline{x}_2$ fulfil the equation

$$\begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} = \underline{P}_{eo}\underline{x} \qquad\qquad (60)$$

(confer equation (37)).

In the following, the first lifting stage 501 is explained, which is the lifting stage corresponding to the lifting matrix $\underline{T_3}$.

5   Let $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ be the output vector of the first lifting stage, for the time being without rounding to integer values, i.e.

$$\begin{bmatrix} \underline{v_1} \\ \underline{v_2} \end{bmatrix} = \underline{T_3} \begin{bmatrix} \underline{x_1} \\ \underline{x_2} \end{bmatrix}$$
                                                                 (61)

10 Using the definition of $\underline{T_3}$ provided by equation (50), equation (61) is re-written as

$$\begin{bmatrix} \underline{v_1} \\ \underline{v_2} \end{bmatrix} = \begin{bmatrix} I_{\frac{N}{2}} & \\ \underline{K_3} & I_{\frac{N}{2}} \end{bmatrix} \begin{bmatrix} \underline{x_1} \\ \underline{x_2} \end{bmatrix}$$
                                                                 (62)

15 Since in this embodiment, a reversible algorithm for integer DCT-IV in this provided, there is included a rounding to integer values. So, according to equation (62), in the first step 506 of the first lifting stage 501, $\underline{x_1}$ is multiplied by $\underline{K_3}$. The result of this multiplication is rounded to integer

20 values in step 507. The rounded values are then added to x_2 in step 508. Thus, the intermediate signal $\underline{z_1}$ fulfils the equation

$$\underline{z_1} = \lfloor \underline{K_3} \cdot \underline{x_1} \rceil + \underline{x_2}$$
                                                                 (63)

25

where $\lfloor * \rceil$ denotes rounding operation.

Since the other four lifting stages 502, 503, 504, 505 of the transforming element illustrated in figure 5, which correspond to the matrices $\underline{T}_2$, $\underline{S}$, $\underline{R}_2$ and $\underline{R}_1$ respectively, have identical structures as the first lifting stage 501,

5   their explanation is omitted. It only has to be noted that in the adding step 509 of the second lifting step 502, $\underline{x}_1$ is, according to the definition of $\underline{T}_2$, multiplied with $-\underline{D}_{N/2}$.

In the following, the lifting stages of the transforming

10   element of the reverse transformation are explained with reference to figure 6.

Figure 6 illustrates the lifting stages of the transforming element for the reverse transformation of the transformation

15   illustrated in figure 5.

In figure 6, the input of the first lifting stage are the two blocks of the digital signal $\underline{y}_1$ and $\underline{y}_2$, $\underline{z}_1$, $\underline{z}_2$ and $\underline{z}_3$ are intermediate signals and $\underline{x}_1$ and $\underline{x}_2$ are output signals

20   corresponding to the first and second block of the digital signal, respectively.

The last lifting stage 605 illustrated in figure 6 is inverse to the first lifting stage 501 illustrated in figure 5. So,

25   in the first step 606 of the last lifting stage 605, $\underline{x}_1$ is multiplied by $\underline{K}_3$. The result of this multiplication is rounded to integer values in step 607. The rounded values are then subtracted from $\underline{z}_1$ in step 608. Thus, the signal $\underline{x}_2$ fulfils the equation

30

$$\underline{x}_2 = \underline{z}_1 - \left\lfloor \underline{K}_3 \cdot \underline{x}_1 \right\rfloor \qquad (64)$$

where $\lfloor * \rceil$ denotes rounding operation.

Since the other four lifting stages 601, 602, 603, 604 of the transforming element illustrated in figure 6, which are
5   inverse to the lifting stages 505, 504, 503 and 502 respectively, have identical structures as the last lifting stage 605, their explanation is omitted. It only has to be noted that after the adding step 609 of the fourth lifting step 604, the result of the adding step 609 is multiplied
10   with $-\underline{D}_{N/2}$ to produce $\underline{x}_1$.

It can be seen that the lifting stages 605, 604, 603, 602 and 601 of figure 6 are inverse to the lifting stages 501 to 505 of figure 5, respectively. Since also the permutation of the
15   input signal corresponding to the matrix $\underline{P}_{eo}$ can be reversed and an according data shuffling stage is comprised by the inverse transforming element, the provided method is reversible, thus, if used in the audio encoder 100 and the audio decoder 200 illustrated in figures 1 and 2, providing a
20   method and an apparatus for lossless audio coding.

An analysis of the number of roundings used in this embodiment is given at the end of the description of the invention.
25
Figure 7 shows the architecture for an image archiving system according to an embodiment of the invention.

In figure 7 an image source 701, for instance a camera,
30   provides an analog image signal. The image signal is processed by a analog-to-digital converter 702 to provide a corresponding digital image signal. The digital image signal is losslessy encoded by a lossless image encoder 703 which

includes a transformation from the time domain to the
frequency domain. In this embodiment, the time domain
corresponds to the coordinate space of the image. The
lossless coded image signal is stored in a storage device
704, for example a hard disk or a DVD. When the image is
needed, the losslessly coded image signal is fetched from the
storage device 704 and provided to a lossless image decoder
705 which decodes the losslessly coded image signal and
reconstructs the original image signal without any data loss.

Such lossless archiving of image signals is important, for
example, in the case that the images are error maps of
semiconductor wafers and have to be stored for later
analysis.

In the following, a further embodiment of the method for the
transformation of a digital signal from the time domain to
the frequency domain and vice versa according to the
invention is explained. This embodiment is preferably used in
the lossless image encoder 703 and the lossless image decoder
705 of the image archiving system illustrated in figure 7.

Figure 8 illustrates an embodiment of the method according to
the invention using DWT-IV as the transformation function.

The DWT-IV of a N-point real input sequence $x(n)$ is defined
as follows:

$$y(m) = \sqrt{\frac{2}{N}} \sum_{n=0}^{N-1} x(n) \sin\left(\frac{\pi}{4} + \frac{(m+1/2)(n+1/2)2\pi}{N} \right) \quad m,n = 0,1,\cdots,N-1 \qquad (65)$$

The transformation matrix $\underline{W_N^{IV}}$ of DWT-IV is given by

$$W_N^{IV} = \sqrt{\frac{2}{N}} \left[\sin\left(\frac{\pi}{4} + \frac{(m+1/2)(n+1/2)2\pi}{N} \right) \right]_{m,n=0,1,\cdots,N-1} \tag{66}$$

According to the process according to the invention, the DWT-IV matrix is factorised into the following form:

$$W_N^{IV} = R_N T P_N \tag{67}$$

R_N is a $N \times N$ rotation matrix defined as

$$R_N = \frac{1}{\sqrt{2}} \begin{bmatrix} I_{N/2} & J_{N/2} \\ -J_{N/2} & I_{N/2} \end{bmatrix} \tag{68}$$

$I_{N/2}$ is the identity matrix of order N/2 (confer equation (29)). $J_{N/2}$ is the counter identity matrix of order N/2 (confer equation (30)).

P_N is a $N \times N$ permutation matrix :

$$P_N = \begin{bmatrix} I_{N/2} & \\ & J_{N/2} \end{bmatrix} \tag{69}$$

T is a $N \times N$ matrix given by

$$T = \frac{1}{\sqrt{2}} \begin{bmatrix} C_{N/2}^{IV} & -C_{N/2}^{IV} \\ C_{N/2}^{IV} D_{N/2} & C_{N/2}^{IV} D_{N/2} \end{bmatrix} \tag{70}$$

where $C_{N/2}^{IV}$ is the DCT-IV matrix of order N/2

$$\underline{C_{N/2}^{IV}} = \sqrt{\frac{2}{(N/2)}}\left[\cos\left(\frac{(m+1/2)(n+1/2)\pi}{(N/2)}\right)\right]_{m,n=0,1,\cdots,N/2-1} \tag{71}$$

$\underline{D_{N/2}}$ is an order N/2 diagonal matrix given by

$$\underline{D_{N/2}} = \begin{bmatrix} 1 & & & & \\ & -1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & -1 \end{bmatrix} \tag{72}$$

According to the process according to the invention, $\underline{R_N}$ and $\underline{T}$ are further factorised into a product of lifting matrices:

$$\underline{T} = \underline{T_1 T_2 T_3} = \begin{bmatrix} I_{N/2} & \\ \underline{K_1} & I_{N/2} \end{bmatrix} \cdot \begin{bmatrix} \underline{D_{N/2}} & \underline{K_2} \\ & I_{N/2} \end{bmatrix} \cdot \begin{bmatrix} I_{N/2} & \\ \underline{K_3} & I_{N/2} \end{bmatrix} \tag{73}$$

where $\underline{K_1} = \left(\sqrt{2}\underline{I_{N/2}} - \underline{C_{N/2}^{IV}}\underline{D_{N/2}}\right)\underline{C_{N/2}^{IV}}$ $\underline{K_2} = \frac{-\underline{C_{N/2}^{IV}}}{\sqrt{2}}$ $\underline{K_3} = \sqrt{2}\underline{C_{N/2}^{IV}}\underline{D_{N/2}} - \underline{I_{N/2}}$,

and

$$\underline{R_N} = \underline{R_1 R_2 R_3} = \begin{bmatrix} I_{N/2} & \\ \underline{H_1} & I_{N/2} \end{bmatrix} \cdot \begin{bmatrix} I_{N/2} & \underline{H_2} \\ & I_{N/2} \end{bmatrix} \cdot \begin{bmatrix} I_{N/2} & \\ \underline{H_3} & I_{N/2} \end{bmatrix} \tag{74}$$

where

$$\underline{H_1} = \underline{H_3} = -\tan(\pi/8)\cdot\underline{J_{N/2}} = \begin{bmatrix} & & -0.414 \\ & -0.414 & \\ & \cdots & \\ -0.414 & & \end{bmatrix} \tag{75}$$

and

$$H_2 = \sin(\pi/4) \cdot J_{N/2} = \begin{bmatrix} & & & 0.707 \\ & & 0.707 & \\ & \ddots & & \\ 0.707 & & & \end{bmatrix} \tag{76}$$

Thus, equation (67) can be written in the form

5   $W_N^{IV} = R_1 R_2 R_3 T_1 T_2 T_3 P_N$ (77)

According to the process according to the invention, the lifting steps are aggregated as far as possible.

10 In this embodiment the lifting matrices R_3 and T_1 can be aggregated to the lifting matrix S:

$$S = R_3 T_1 = \begin{bmatrix} I_{N/2} & \\ H_3 + K_1 & I_{N/2} \end{bmatrix} \tag{78}$$

15 From (77) and (78), the following factorisation formula for the DWT-IV matrix is obtained:

$$W_N^{IV} = R_1 R_2 S \, T_2 T_3 P_N \tag{79}$$

20 Equation (79) indicates that the transforming element for the integer DWT-IV transform according to the invention comprises five lifting stages.

Further, the transforming element comprises a data shuffling
25 stage corresponding to the permutation matrix P_N. The data shuffling stage rearranges the components order in each input data block. According to P_N, the input data vector is rearranged in the following way: the first half of vector

remains unchanged; the second half of vector is flipped,
i.e.:

$$\underline{P_N}\underline{x} = \underline{P_N}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N/2} \\ x_{N/2+1} \\ \vdots \\ x_{N-1} \\ x_N \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N/2} \\ x_N \\ x_{N-1} \\ \vdots \\ x_{N/2+1} \end{bmatrix} \tag{80}$$

5

In figure 8, the input of the first lifting stage are the two
blocks of the digital signal $\underline{x_1}$ and $\underline{x_2}$, $\underline{z_1}$, $\underline{z_2}$ and $\underline{z_3}$ are
intermediate signals and $\underline{y_1}$ and $\underline{y_2}$ are output signals
corresponding to the first and second block of the digital

10   signal, respectively.

The input of the transforming element $\underline{x}$ and the two input
blocks of the first lifting stage of the transforming element
$\underline{x_1}$ and $\underline{x_2}$ fulfil the equation

15

$$\begin{bmatrix} \underline{x_1} \\ \underline{x_2} \end{bmatrix} = \underline{P_N}\underline{x} \tag{81}$$

In the following, the first lifting stage 801 is explained,
which is the lifting stage corresponding to the lifting

20   matrix $\underline{T_3}$.

Let $\begin{bmatrix} \underline{v_1} \\ \underline{v_2} \end{bmatrix}$ be the output vector of the first lifting stage, for

the time being without rounding to integer values, i.e.

$$\begin{bmatrix} \underline{v_1} \\ \underline{v_2} \end{bmatrix} = \underline{T_3} \begin{bmatrix} \underline{x_1} \\ \underline{x_2} \end{bmatrix} \qquad (82)$$

Using the definition of $\underline{T_3}$ provided by equation (73),

equation (82) is re-written as

5

$$\begin{bmatrix} \underline{v_1} \\ \underline{v_2} \end{bmatrix} = \begin{bmatrix} I_{\frac{N}{2}} & \\ \underline{K_3} & I_{\frac{N}{2}} \end{bmatrix} \begin{bmatrix} \underline{x_1} \\ \underline{x_2} \end{bmatrix} \qquad (83)$$

Since in this embodiment, a reversible algorithm for integer

DWT-IV in this provided, there is included a rounding to

10 integer values. So, according to equation (83), in the first

step 806 of the first lifting stage 801, $\underline{x_1}$ is multiplied by

$\underline{K_3}$. The result of this multiplication is rounded to integer

values in step 807. The rounded values are then added to x_2

in step 808. Thus, the intermediate signal $\underline{z_1}$ fulfils the

15 equation

$$\underline{z_1} = \left\lfloor \underline{K_3} \cdot \underline{x_1} \right\rfloor + \underline{x_2} \qquad (84)$$

where $\lfloor * \rfloor$ denotes rounding operation.

20

Since the other four lifting stages 802, 803, 804, 805 of the

transforming element illustrated in figure 8, which

correspond to the matrices $\underline{T_2}$, $\underline{S}$, $\underline{R_2}$ and $\underline{R_1}$ respectively,

have identical structures as the first lifting stage 801,

25 their explanation is omitted. It only has to be noted that in

the adding step 809 of the second lifting step 802, $\underline{x_1}$ is,

according to the definition of $\underline{T_2}$, multiplied with $\underline{D_{N/2}}$.

38

In the following, the lifting stages of the transforming
element of the reverse transformation are explained with
reference to figure 9.

5   Figure 9 illustrates the lifting stages of the transforming
element for the reverse transformation of the transformation
illustrated in figure 8.

In figure 9, the input of the first lifting stage are the two
10 blocks of the digital signal $\underline{y}_1$ and $\underline{y}_2$, $\underline{z}_1$, $\underline{z}_2$ and $\underline{z}_3$ are
intermediate signals and $\underline{x}_1$ and $\underline{x}_2$ are output signals
corresponding to the first and second block of the digital
signal, respectively.

15 The last lifting stage 905 illustrated in figure 9 is inverse
to the first lifting stage 801 illustrated in figure 8. So,
in the first step 906 of the last lifting stage 905, $\underline{x}_1$ is
multiplied by $\underline{K}_3$. The result of this multiplication is
rounded to integer values in step 907. The rounded values are
20 then subtracted from $\underline{z}_1$ in step 908. Thus, the signal $\underline{x}_2$
fulfils the equation

$$\underline{x}_2 = \underline{z}_1 - \left\lfloor \underline{K}_3 \cdot \underline{x}_1 \right\rfloor \tag{85}$$

25 where $\lfloor * \rfloor$ denotes rounding operation.

Since the other four lifting stages 901, 902, 903, 904 of the
transforming element illustrated in figure 9, which are
inverse to the lifting stages 805, 804, 803 and 802
30 respectively, have identical structures as the last lifting
stage 905, their explanation is omitted. It only has to be
noted that after the adding step 909 of the fourth lifting

step 904, the result of the adding step 909 is multiplied with $\underline{D_{N/2}}$ to produce $\underline{x_1}$.

It can be seen that the lifting stages 905, 904, 903, 902 and 901 of figure 9 are inverse to the lifting stages 801 to 805 of figure 8, respectively. Since also the permutation of the input signal corresponding to the matrix $\underline{P_N}$ can be reversed and an according data shuffling stage is comprised by the inverse transforming element, the whole provided method is reversible, thus, if used in the lossless image encoder 703 and the lossless image decoder 705 illustrated in figure 7, providing a method and an apparatus for lossless image coding.

Although in the explained embodiments, the method according to the invention for DCT-IV was used for audio coding and the method according to the invention for DWT-IV was used for image coding, the method according to the invention for DCT-IV can as well be used for image coding, the method according to the invention for DWT-IV can as well be used audio coding and both can be used as well for the coding of other digital signals, such as video signals.

Considering the equations (63) and (64) one sees that there are $N/2$ roundings in each lifting stage. Therefore, considering equation (59), one sees that the total rounding number for the transforming element for the DCT-IV algorithm according to the invention is five times $N/2$, that is $2.5N$, which is significantly lower than $Nlog_2N$ according to the state of the art.

Again considering equation (59), one sees that the majority of computation power is used in the four $N/2$ point DCT-IV

40

subroutines corresponding to a multiplication with $C_{N/2}^{IV}$, when
N is a large value, e.g. $N = 1024$. Because the floating-point
DCT-IV can be calculated using two half-length DCT-IV plus
pre- and post-rotations according to the equations (47) and
5   (49), the arithmetic complexity of the proposed integer DCT-
IV is roughly estimated to be twice that of the floating-
point DCT-IV.

A similar conclusion can be drawn for the integer DWT-IV
10   transformation function.

In the following, another embodiment is explained wherein the
transformation is a discrete Fourier transformation.

15   The transformation matrix $\underline{F}_N$, which is the transformation
matrix of the normalized FFT of order N is given by:

$$\underline{F}_N = \sqrt{\frac{1}{N}} \left[\exp\left(\frac{-j2\pi mn}{N}\right) \right] \qquad (86)$$
$$m, n = 0, 1, \cdots, N - 1$$

20   where transform size N is even .

Following the decimation-in-time radix-2 FFT algorithm, $\underline{F}_N$
can be decomposed in the following way:

$$25 \qquad \underline{F}_N = \begin{bmatrix} \underline{I}_{N}/2\big/\sqrt{2} & \underline{I}_{N}/2\big/\sqrt{2} \\ \underline{I}_{N}/2\big/\sqrt{2} & -\underline{I}_{N}/2\big/\sqrt{2} \end{bmatrix} \begin{bmatrix} \underline{I}_N/2 & \\ & \underline{W} \end{bmatrix} \begin{bmatrix} & \underline{F}_N/2 \\ \underline{F}_N/2 & \end{bmatrix} \underline{P}_{eo} \qquad (87)$$

Where $\underline{P}_{eo}$ is, as above, an even-odd matrix, i.e, a
permutation matrix reordering the components of a vector

$$\underline{x} = \begin{bmatrix} x(0) \\ \vdots \\ x(N-1) \end{bmatrix}$$

by separating the components which correspond to even indices
from the components which correspond to odd indices, such

5   that

$$\begin{bmatrix} x(0) \\ x(2) \\ \vdots \\ x(N-2) \\ x(1) \\ x(3) \\ \vdots \\ x(N-1) \end{bmatrix} = \underline{P}_{eo} \begin{bmatrix} x(0) \\ \vdots \\ x(N-1) \end{bmatrix} \qquad (88)$$

Let $\underline{E}_{N/2}$ be the transform matrix of the normalized FFT of

10   order $N/2$.

Let further $\underline{W}$ be a diagonal matrix given by

$$\underline{W} = \begin{bmatrix} W_N^0 & & & \\ & W_N^1 & & \\ & & \ddots & \\ & & & W_N^{N/2-1} \end{bmatrix}$$

15

where $W_N = e^{-j2\pi/N}$.

$\underline{I}_{N/2}$ denotes, as above, the identity matrix of order $N/2$.

42

In Equation (87), the first matrix from left is an even-odd
matrix $\underline{P}_{eo}$, which only reorders the components in the input
vector.

5   In Equation (87), the second matrix from left can be
factorized into three lifting matrices in the following way:

$$
\begin{bmatrix} & \underline{F}_{N/2} \\ \underline{F}_{N/2} & \end{bmatrix} = \begin{bmatrix} \underline{I}_{N/2} & \\ -\underline{Q}\,\underline{F}_{N/2} & \underline{I}_{N/2} \end{bmatrix} \begin{bmatrix} -\underline{Q} & \underline{F}_{N/2} \\ & \underline{I}_{N/2} \end{bmatrix} \begin{bmatrix} \underline{I}_{N/2} & \\ \underline{F}_{N/2} & \underline{I}_{N/2} \end{bmatrix}
$$

(89)

10

where $\underline{Q}$ is an order-$N/2$ permutation matrix given by

$$
\underline{Q} = \begin{bmatrix} 1 & 0 \\ 0 & \underline{J}_{N/2-1} \end{bmatrix}
$$

15   and $\underline{J}_{N/2-1}$ is the order $N/2-1$ counter index matrix.

In Equation (87), the third matrix from left is a counter-
diagonal matrix, which merely multiplies half of the
components in the input vector by a complex number residing
20   on the unit circle.

This can be interpreted as a rotation on the complex plane.

Let $x = x_r + jx_i$ be such a component in the input vector.

25

Let further

$$
W_N^k = e^{-j2k\pi/N} = \cos(2k\pi/N) - j\sin(2k\pi/N) = c_k - js_k \text{ be the}
$$

complex number, i.e. the component of $\underline{W}$, which is multiplied
30   with x when the input vector, after it has been multiplied

with the first matrix from the right and the second matrix on
the right of equation (87), is multiplied with $\underline{W}$.

The result is

5 $y = y_r + jy_i = \underline{W}_N^k x = (c_k x_r + s_k x_i) + j(c_k x_i - s_k x_r)$, which equals
x rotated counter-clockwise by $2k\pi/N$ radius on the complex
plane. Such a rotation can be factorized into three lifting
steps as follows:

10
$$\begin{bmatrix} y_r \\ y_i \end{bmatrix} = \begin{bmatrix} c_k & s_k \\ -s_k & c_k \end{bmatrix}\begin{bmatrix} x_r \\ x_i \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{c-1}{s} & 1 \end{bmatrix}\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{c-1}{s} & 0 \end{bmatrix}\begin{bmatrix} x_r \\ x_i \end{bmatrix} \qquad (90)$$

In Equation (87), the fourth from the right can be factorized
into three lifting matrices in the following way:

15
$$\begin{bmatrix} \underline{I}_N/2/\sqrt{2} & \underline{I}_N/2/\sqrt{2} \\ \underline{I}_N/2/\sqrt{2} & -\underline{I}_N/2/\sqrt{2} \end{bmatrix} =$$
$$\begin{bmatrix} (\sqrt{2}-1)\,\underline{I}_N/2 & \underline{I}_N/2 \\ \underline{I}_N/2 & \end{bmatrix}\begin{bmatrix} \underline{I}_N/2 & -1/\sqrt{2}\,\underline{I}_N/2 \\ & \underline{I}_N/2 \end{bmatrix}\begin{bmatrix} & \underline{I}_N/2 \\ (\sqrt{2}-1)\,\underline{I}_N/2 & \underline{I}_N/2 \end{bmatrix}$$
$$(91)$$

The decimation-in-frequency radix-2 FFT algorithm is merely a
transposition of the decimation-in-time radix-2 algorithm in
20 Equation (87).

Thus, the above procedures also apply to factorising the FFT
matrix $\underline{F}_N$ in the decimation-in-frequency way.

25 Using the factorisation of the right hand side of equation
(87) in lifting matrices by generating a lifting stage
corresponding to each lifting matrix a transforming element
is determined.

Since it has been already detailed above how lifting stages
are generated according to lifting matrices and in this
embodiment this works analogously to above, the explanation
5   is omitted here.

45

In this document, the following documents are herein
incorporated by reference:

[1] H. S. Malvar, "Signal Processing with Lapped Transforms"
5   *Artech House*, 1992

[2] R. Geiger, T. Sporer, J. Koller, K. Brandenburg, "Audio
       Coding based on Integer Transforms" *AES 111th Convention*,
       New York, USA, Sept. 2001
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